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# Optimization of Aerospace Structures under Uncertainty using an Iterative Distribution Evolutionary Algorithm

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An Iterative Differential Evolutionary Algorithm is proposed for the optimized identification of sets of parameter values for a system whose analysis is very time demanding and for which it is difficult to identify and define the optimum set. The algorithm is an evolutionary technique and reduces the number of evaluations required for both numerical and experimental systems thanks to an iterative procedure. It exploits geometrical considerations to distribute the points to be investigated in the considered parameter space, and SVD/metamodeling based techniques to further decrease the computational burden. The developed strategy aims to minimize the probability of failure, guaranteeing a reliable optimum, providing an understanding of the acceptable range of uncertainties and keeping robustness. The approach is validated considering the optimization of landing gear designs minimizing the probability of occurrence of shimmy phenomena during ground manoeuvres.

## Nomenclature

$B, Bm$	track of the main assembly and distance between the nose and the axis of the main assembly
$d$	direction of interest
$\mathbf{F}^*$	set of tangency parameters
$\bar{\mathbf{F}}_{\text{opt}}$	optimum nominal parameter value
$g$	limit state function
$L$	Lift
$LoI$	locus of interest
$P_{max_i}$	maximum range of variation for the $i$ th considered parameters
$Tol_\epsilon, Tol_p$	tolerances adopted in the algorithm
$\bar{W}$	vertical load acting on the landing gear structure
$W$	weight of the aircraft
$\mathbf{x}$	vector of design parameters

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## I. Introduction

The adoption of optimization strategies in a design process is highly recommended in all engineering fields. The optimization process often faces multi-objective problems and in the field of structural reliability the minimization of occurrence of limit-state or constraints violations is analysed.<sup>1</sup> The defined objectives can be conflicting and so the optimum solution is not unique and may not optimize each individual objective but is the result of a compromise. Pareto frontier analyses is often exploited to find the optimum compromise.<sup>2</sup> Pareto frontier analysis was first introduced by V. Pareto in the economic field but it has often been applied in the engineering field,<sup>3</sup> such as the landing gear design process.<sup>4</sup>

The minimization of the landing gear weight has been considered as the main objective function also in structural optimization of UAV landing gears<sup>5,6</sup>. Landing gear design can also be the result of structural optimization based on fatigue life<sup>7</sup> or on both fatigue life and energy absorption during landing and take off.<sup>8</sup> Most of the time multi-body systems and/or finite element methods (FEM) have been adopted to depict the actual complexity characterizing the aeronautical system of interest<sup>5-9</sup>. Moreover, depending on the task of the analysis, a metamodel can be used to speed up the process and/or to construct the optimization strategy itself.<sup>7</sup>

In the optimization process, uncertainty in the system has not been always considered and the process is developed adopting a deterministic analysis<sup>3-5</sup> or sometimes introducing a safety factor.<sup>6</sup> The consideration of uncertainties in the development and improvement of such a strategy has recently become a particular research of interest. In fact, the industrial companies themselves are aware that a deterministic approach, with the application of a safety factor, involves an over or under designed system.

The consideration of uncertainties increases the difficulties in determining an optimum nominal design. This is particularly true for systems whose numerical model is computationally expensive to be simulated. Robust design optimization (RDO)<sup>10</sup> and reliability-based design optimization (RBDO)<sup>11</sup> are the two techniques commonly adopted to deal with the optimization of performance of systems under uncertainty<sup>12,1</sup>.

In the presence of a very computationally expensive numerical model, nonlinear behaviour and multi-objective problems, the traditional RDO and RBDO are not always suitable because of two main problems: the prohibitive computational cost and the neglect of higher-order moments common for the RDO and RBDO techniques. Alternative optimization strategies have been proposed to overcome these problems, for instance aggressive design procedures<sup>13,14</sup>. The difficulty in such an approach, is that, depending on the problem, it is not assured that the desired target is matched ‘close’ enough or that the computational effort is actually reduced. In this scenario, the necessity arises to develop a method that can assure reliability for an engineering structure whose analysis is time demanding even in the absence of uncertainty.

In this paper, a novel optimization method (Iterative Distribution Evolutionary Algorithm - IDEA) is presented and can be classified as an evolutionary algorithms.<sup>15-17</sup> It is an iterative procedure that aims to robustly minimize the probability of failure and to provide an understanding of the acceptable range of uncertainties exploiting geometrical considerations to distribute the points to be investigated in the considered parameter space, and SVD/metamodelling based techniques.<sup>18-22</sup> The optimization strategy is conceived in order to solve problems that are very time demanding and for which it is difficult (and expensive) to determine derivatives and to identify and define the optimum set of parameters. The reduction in computational time is both in terms of number of cases to be analysed directly through experiments or run of numerical models and in the lack of a need to compute gradients. The completeness of the analysis is fulfilled thanks to the inclusion of methodologies to perform sensitivity analysis (SA) and propagate uncertainties in the system. The validation considers the optimization of landing gear design to avoid shimmy phenomena during ground manoeuvre.

## II. Methodology

The optimization of a system requires to first identify the dynamics that influence the most the responses of interest. To this end techniques already adopted by the author<sup>20,22,23</sup> to perform sensitivity analysis (SA) and uncertainty quantification (UQ) have been adopted. Sobol indexes are used for performing SA and SVD/metamodelling based techniques for the uncertainty propagation. Such a techniques allow a better understanding of phenomena of interest and of the influence of input factors in order to then be able in improving the design of engineering systems. Once the input factors most influential to the response of interest are identified, then the optimization in terms of these factors can be performed. An optimization

problem is formulated as it follows. A vector of values  $x_1, x_2, \dots, x_N$  for an input factor vector  $\mathbf{x}$  is sought in order to minimize the defined objective function  $f(\mathbf{x})$ .

The proposed optimization strategy is considering the nominal values of the selected design factors as uncertain. However, there is not any kind of limitation in the design/uncertain factors that can be selected. The strategies are conceived in order to solve problems that are very time demanding and for which it is difficult (and expensive) to determine derivatives and to identify and define the optimum set of parameters. The reduction in computational time is both in terms of number of cases to be analyzed directly through experiments or run of numerical models and in the lack of a need to compute gradients. The completeness of the analysis is fulfilled thanks to the inclusion of methodologies to perform SA and propagate uncertainties in the system.

The proposed novel optimization methods (Iterative Distribution Evolutionary Algorithm - IDEA) can be included among the evolutionary algorithms.<sup>15–17</sup> Evolutionary Algorithms (EAs) are population-based metaheuristic optimization algorithm; they are exploring the set of possible solutions for sufficient set of solutions following mechanism inspired by biological evolution, for instance reproduction, generation, mutation, recombination and selection. The optimization considers the input factors as individuals belonging to populations that are generated through mutation and recombination and that can be subjected to mutation. EAs techniques are fascinating since they can be adopted whatever the problem of interest, since no assumptions are made.<sup>24</sup> However, the determined solution is numerical and is an approximation of the unknown optimal one. In this scenario, it is important to remark that in engineering problems an optimum result can be difficult to be identified and is not uniquely defined. Thus, in such a scenario evolutionary algorithms are sensible to be adopted. The proposed optimization algorithm is a procedure that aims to minimize the probability of failure without directly computing it since the unfeasible computational cost in obtaining the probability density function of the quantity of interest. IDEA aims to determine the range of variation for the input factors for which the uncertain boundary is tangent to the limit condition from the reliable area. It also provides an understanding of the acceptable range of uncertainties exploiting geometrical considerations to distribute the points to be investigated in the considered parameter space. The proposed technique is composed of three phases, the first two are adopted to identify the range of variation to start the optimization process. In fact, the starting step for the iterative procedure is to identify the set of values for the parameters  $\mathbf{F}^*$  for which the locus of interest *LoI* is tangent to the defined limit-state function  $g$  or for which the distance along the direction of interest  $d$  for which the probability of failure is highest is a minimum, given a starting set for the parameters  $\mathbf{F}$ . For the sake of clarity Fig. 1 shows an example of a generic problem in which the direction of interest is connecting the origin of the space defined by the quantities of interest (QoI), i.e. the quantities that describe the locus of interest and limit-state function, and the point in which the limit state function is crossed.

The stated phases are:

- First phase: Preparation
- Second phase: Data Collection
- Third phase: Iterative Process

In the following sections, three phases are presented and then in section IV the specialization for the case of interest and the obtained results are discussed.

**FIRST PHASE: PREPARATION** The objectives of the optimization are established including possible acceptable tolerances. In particular, the limit state function  $g$  that delimits the failure region has to be defined. For the application here considered, the objective function is the variation of the vertical load on the landing gear as the forward velocity changes during a static maneuver on the ground of an illustrative aircraft in equilibrium conditions. The stated variation can be evaluated employing equilibrium equations. The explicit expression of the stated variation is given by

$$\overline{W} = \frac{B_m}{B}(W - L) \quad L = \frac{1}{2}\rho V^2 S C_L \quad (1)$$

where  $B_m$  is the track of the main assembly,  $B$  is the distance between the nose and the axis of the main assembly,  $W$  is the weight of the aircraft,  $L$  is the lift (Figure 2).

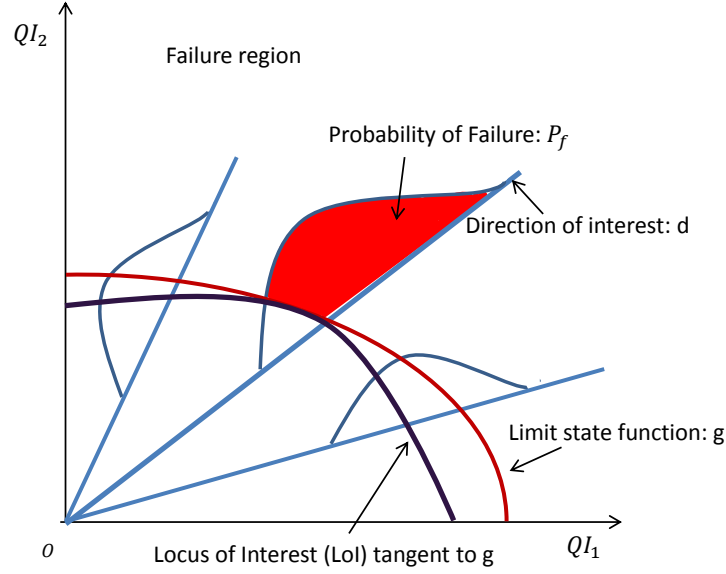


Figure 1: Example of how to detect the direction of interest  $d$  looking at the probability of failure.

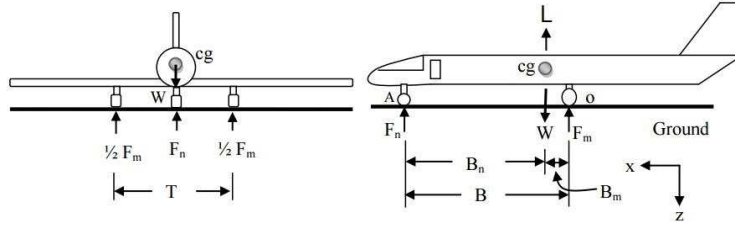


Figure 2: Quantities needed for writing equilibrium equations.

Moreover, in the presence of a system with a lot of parameters, like in a landing gear system, sensitivity analysis needs to be performed to detect the most influential ones for the considered objectives in the optimization process. Sobol indices<sup>25,26</sup> are adopted. Having identified the parameters to be considered during the optimization process, the maximum negative range of variation  $P_{max_{i_{neg}}}$  and positive one  $P_{max_{i_{pos}}}$  for the  $i$ th considered parameter ( $i = 1 \dots N$ ) are defined. In case a symmetric variation of the parameter of interest is adopted, then  $P_{max_{i_{neg}}}$  is equal to  $P_{max_{i_{pos}}}$  and the maximum percentage is  $P_{max_i}$ . Here, this variation has been adopted.

**SECOND PHASE: DATA COLLECTION** The quantities of interests (QoIs), those that describe the locus of interest and limit-state function, are evaluated for a suitable number of points in the parameter space by directly running the numerical model or doing experiments. These are needed to train surrogate models adopted in the SVD based methodology.<sup>18-22</sup> Using the SVD/metamodelling based methodology can reduce by 95% the time consumption due to the investigation over the parameters space to look for the set of nominal values  $\mathbf{F}^*$  for which the stated tangency occurs. Knowing  $\mathbf{F}^*$ , the interval  $(x_{j,low}, x_{j,upp})$  of interest for each  $j$ th design parameter can be defined (eq. (2)).

$$\left[ F_i^* \cdot \frac{1 - P_{max_{i_{neg}}}}{1 + P_{max_{i_{pos}}}}, F_i^* \cdot \frac{1 + P_{max_{i_{pos}}}}{1 - P_{max_{i_{neg}}}} \right] \quad (2)$$

The range of variation defined in eq. 2 is the same for both the optimization strategies, but the definition comes from considerations related to the IDEA technique, i.e. the point  $\mathbf{F}^*$  need to be internal to the

optimum uncertain interval of variation identified at the end of the IDEA process and that is around the optimum nominal value  $\bar{\mathbf{F}}_{opt}$ . In fact, the point  $\mathbf{F}^*$  is the one for which the locus of interest is tangent to the limit-state function. The steps to obtain the interval shown in eq. (2) are now explained. The possible upper and lower bounds for the optimum nominal value, given that the point  $\mathbf{F}^*$  needs to be internal to the final optimum uncertain interval of variation, are shown in eqs. (3) and (4).

$$\bar{\mathbf{F}}_{opt_{upp}}(1 - P_{max_{i_{neg}}}) = F_i^* \quad (3)$$

$$\bar{\mathbf{F}}_{opt_{low}}(1 + P_{max_{i_{pos}}}) = F_i^* \quad (4)$$

Considering the maximum possible interval of variation for the lower or upper nominal value  $\bar{\mathbf{F}}_{opt}$  (eq. (5)) and substituting the lower and upper bounds using eqs. (3) and (4), the expression in eq. (2) is obtained.

$$[\bar{\mathbf{F}}_{opt_{low}}(1 - P_{max_{i_{low}}}), \bar{\mathbf{F}}_{opt_{upp}}(1 + P_{max_{i_{upp}}})] \quad (5)$$

It is worth noticing that since the analyses are performed numerically, the tangency can be defined as the state for which the locus of interest  $LoI$  is the nearest one to the defined limit function  $g$  along the direction of interest  $d$  previously identified. Mathematically, this can be expressed as  $\mathbf{F}^* := \mathbf{F} | (dist(g - LoI(\mathbf{F}^*)_d) = \min(dist(g - LoI(\mathbf{F})_d))$ .

At this point, the user can also decide to consider more than one set  $\mathbf{F}^*$  to continue in the optimization process and eventually pick the best set according to other requirements, such as robustness.

**THIRD PHASE: ITERATIVE PROCESS** Figure 3 presents the flow chart describing the last phase, which is the iterative one. Dynasys stands for the Dynamical System Toolbox,<sup>27</sup> the Matlab version of AUTO.

The third phase is the iterative part that has evolutionary characteristics. A general evolutionary algorithm has three main steps: generation, mutation and selection.<sup>15–17</sup> Each generation consists of separate selection and mutation steps performed iteratively. In the IDEA the generation is the region-hypercube of interest identified for each set  $\mathbf{F}^*$  the user wants to use; this hypercube has as many dimensions as the number of design factors and each value of the design factors is delimited by the defined interval (eq. (2)). Moreover, for each generation a full factorial design is considered to define sampling points in which to evaluate the objective function. This consists of generating a well structured sampling plane that has the aim to not exclude any values for a specific parameter that could match desired requirements for some precise values of other parameters. For each generation and each point in the full factorial sampling plane, the QoI need to be evaluated. Thanks to the surrogate models already trained, a saving in time can be achieved. The surrogate models are used to evaluate the QoI at the points that are in the range considered in the first phase. Depending on the obtained QoI the relative points are divided into two groups: positive and negative. The positive points are those for which the loci identified by the QoI are not in the failure region or if they are, the defined tolerance  $Tol_\epsilon$  is fulfilled such that

$$dist(g - LoI(\mathbf{F}_G)_d) - dist(g - LoI(\mathbf{F}_G^*)_d) \leq Tol_\epsilon \cdot dist(g - LoI(\mathbf{F}_G^*)_d) \quad (6)$$

where  $G$  is used here to emphasize belonging to a particular generation.

All the other points are negative and always present since the optimization process is considered for problems that do not have an acceptable probability of failure.

The mutation consists of a subdivision of the hypercube along particular directions such that all the new hypercubes contain the point  $\mathbf{F}_G$ . The directions for the subdivisions are identified by the negative set of points, and in particular by the values assumed by the parameters at such a point. The directions are identified by varying all the parameters at the negative point but one. The number of the directions for each negative point are equal to the dimension of the hypercube, i.e. the number of design parameters. Figure 4 clarifies the procedure considering an hypercube in 2D (the black rectangle) and just one negative point (the yellow point). The black rectangle is always the same initial hypercube and all the possible subdivisions are shown. The blue and red rectangles are those to be neglected or considered to improve the optimization further. At the end of the mutation step, no hypercube with negative points inside should be present.

The selection step consists of sorting the hypercube in a descending order in terms of the volume and evaluating the QoI at the points not in the range considered in phase one. This selection can only be done by directly running the numerical model or using experimental results. Finally, the hypercube that does not have a tolerance greater than  $Tol_p$  of points for which the loci of interest are intersecting the limit state

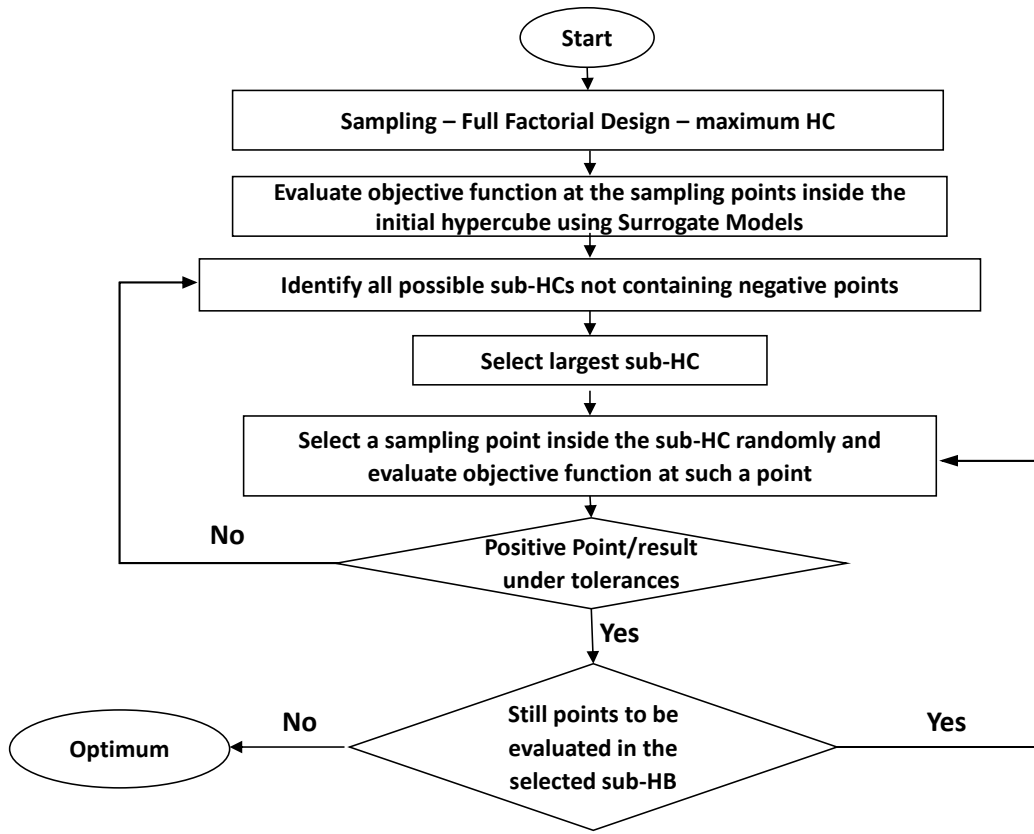


Figure 3: Flow chart describing the iterative phase characterizing the Iterative Distribution Evolutionary Algorithm.

function is selected, having accepted also the tolerance shown in eq. (6).  $Tol_p$  is defined as the percentage of the number of negative points acceptable in the hypercube with respect to the total number of points belonging to the considered hypercube. The optimum set of values is given by the mean point in the hypercube.

**VALIDATION PHASE** The optimum sets of values determined for each generation can be compared, performing uncertainty quantification (UQ) for each of them and looking at the results. This comparison can be accomplished either using just the points belonging to the optimum hypercube or adopting SVD based methods. The desired optimum set is the one that provides the minimum probability of failure (high reliability) and the least width of PDF (high robustness) or a suitable defined compromise of these properties. Once the optimum is found, the nominal value and percentage variation can be defined as preferred, i.e. assuming a symmetric or asymmetric distribution.

### III. Test cases

The test cases of the proposed algorithm are the identification of parameter values for landing gear systems to avoid shimmy phenomena during ground manoeuvre. In this section, the adopted test based models is briefly presented.

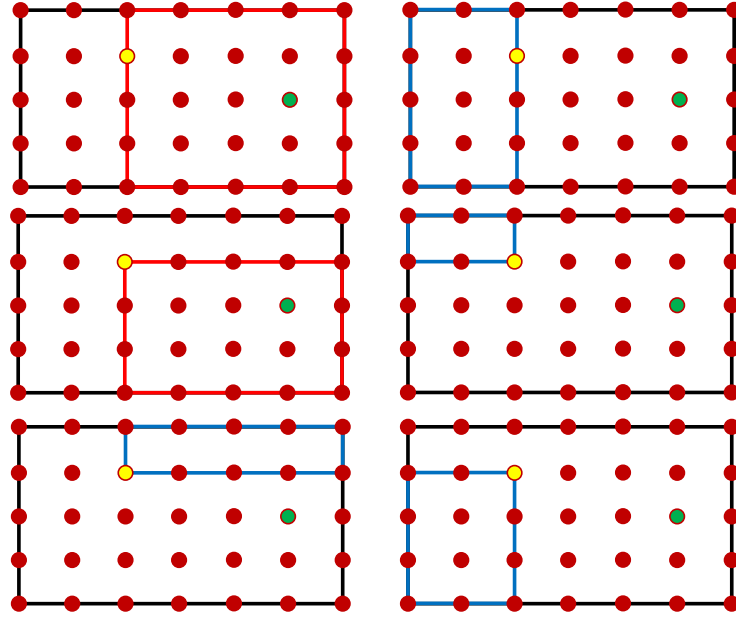


Figure 4: Example of the mutation strategy. The green point is the one related to the set  $\mathbf{F}^*$ . The yellow point belongs to the negative set. The black rectangle is the initial hypercube in 2D, the blue rectangle is the one that does not need further analysis. The red rectangle is the one to improve the optimization further.

### A. Landing gear model

The landing gear model, considered to validate the proposed algorithm, is a multi-body system modeled in LMS Virtual.Lab Motion.<sup>28</sup> Numerical models have been considered, due to the possibilities in describing more in details the system of interest. In fact, they are adopted in industry for static and dynamic analyses in order to limit the assumptions adopted to describe the dynamics of the landing gear model.

The multi-body landing gear model (Fig. 5) is a dual-wheel system and, considering all the adopted constraints without considering the tyre model, it has 9 degrees of freedom and 18. The bodies and adopted joints are shown and explained in Fig. 5. In particular, the joint between Main 1 and Main 2 has been introduced in order to simulate spring-back/spin-up phenomena, the relative degree of freedom is out-of-plane and the relative dynamics is described by second order differential equations in terms of  $\beta$ . Moreover, a nonlinear shock absorber has been modeled and introduced as a nonlinear force acting between the Main 2 and Main 3 bodies and a tyre model based on Pacejka formulation has been developed. The tyre model adds two degrees of freedom for each wheel since it models both the lateral and longitudinal tyre dynamics adopting one order differential equation for each dynamic. The authors have already discussed and shown the importance of including the longitudinal slip/wheels dynamics in a stability analysis.<sup>29</sup>

The stability of both the landing gear systems has been investigated using a bifurcation analysis<sup>30–32</sup> and in particular continuation methods, coupling the models with a continuation software.<sup>20,21,33</sup> The adopted software is the Dynamical System Toolbox,<sup>27</sup> the Matlab version of AUTO. A key benefit of using such an analysis is that there is no need to resort to expensive blind Monte-Carlo simulations that are required when solely using a numerical integration route. The Dynamical System toolbox integrates AUTO into Matlab via mex functions to perform bifurcation analysis of dynamical systems for which an analytic description is available or that are modelled in software able to interface with Matlab. Whatever is the system of interest and the adopted model, Figure 6 presents a flow chart that explains how the continuation analysis is performed using AUTO. At the beginning the bifurcation parameters ( $\mathbf{P}$ ) and the independent states ( $\mathbf{U}$ ) of the system to be considered in order to investigate the stability of the system of interest need to be selected. Then, an equilibrium solution for the system needs to be determined in particular in terms of the selected bifurcation parameters and independent states. These values are used to start the continuation analysis. In order to describe how the continuation works, sub-indexes  $i$  and  $k$  are here adopted. The stated equilibrium solution is given by the set  $(\mathbf{P}_i, \mathbf{U}_{ik}) = (\mathbf{P}_0, \mathbf{U}_{00})$  for  $i = k = 0$ . The index  $i$  stands for each step of the



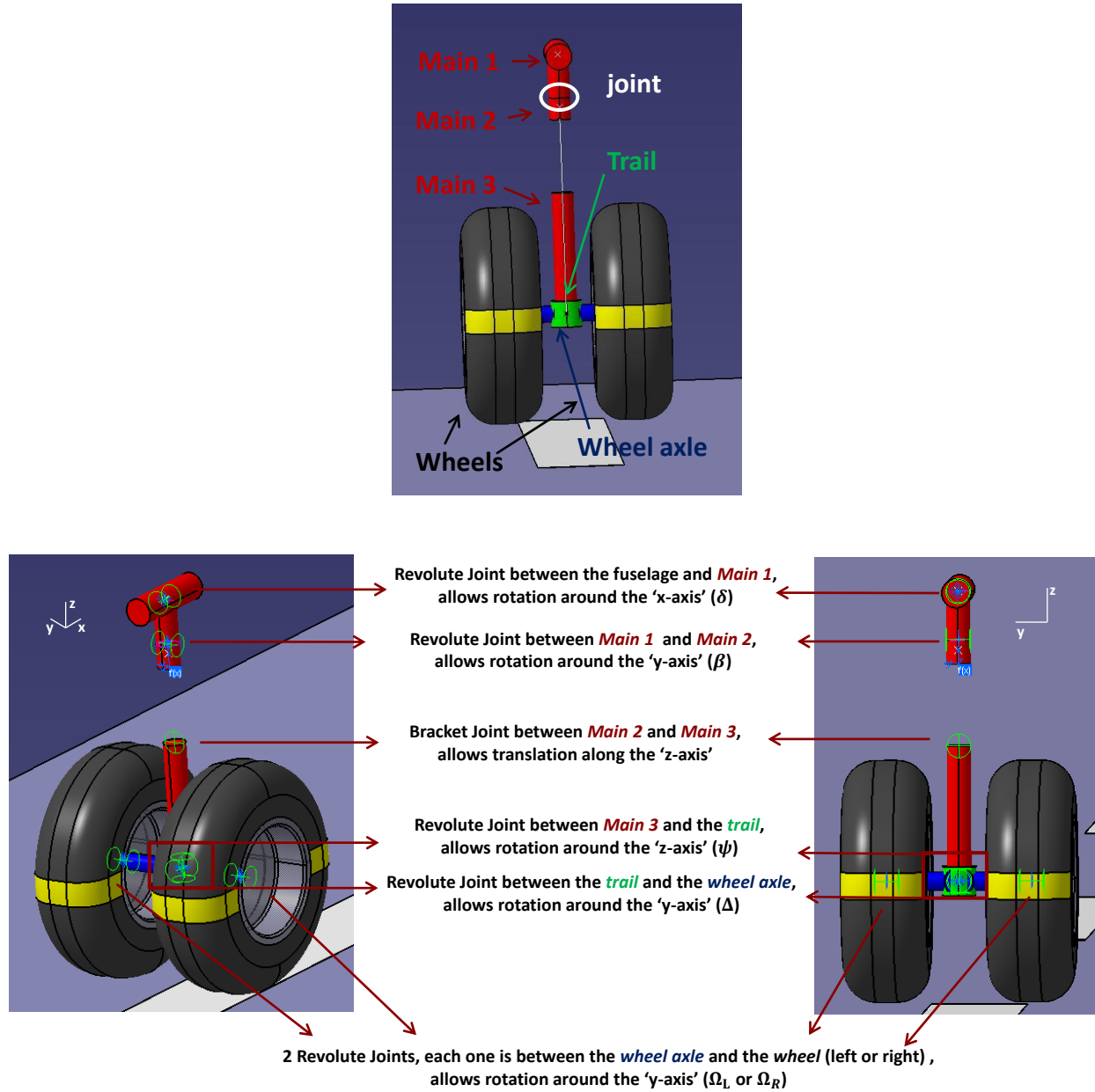


Figure 5: Landing Gear modeled in LMS Virtual.Lab Motion

continuation analysis, the index  $k$  stands for the steps needed to AUTO to determine the correct set of values given to the independent states ( $\mathbf{U}_i$ ), i.e. the values of the independent states ( $\mathbf{U}$ ) such that a steady state solution is found for the considered bifurcation parameter values ( $\mathbf{P}_i$ ). At the end, a set of bifurcation parameters and states ( $\mathbf{P}_i, \mathbf{U}_{ik}$ ) in the range of interest for which the system is in steady state (equilibrium and periodic branch - LCO) can be determined together with the stability of the system along the branches. Moreover, occurrence of bifurcations, i.e. sudden 'qualitative' or topological change of a system due to a small smooth change made to the parameter values (the bifurcation parameters) can be detected. Finally, if self-excited oscillation (Limit Cycle Oscillations such as shimmy or flutter) occurs then a characterization (amplitude, period and steady state response) can be performed.

If a multi-body model is adopted then it is needed a coupling between a multi-body software and one that is able to perform bifurcation analysis. In the present application, the continuation analysis is adopted to

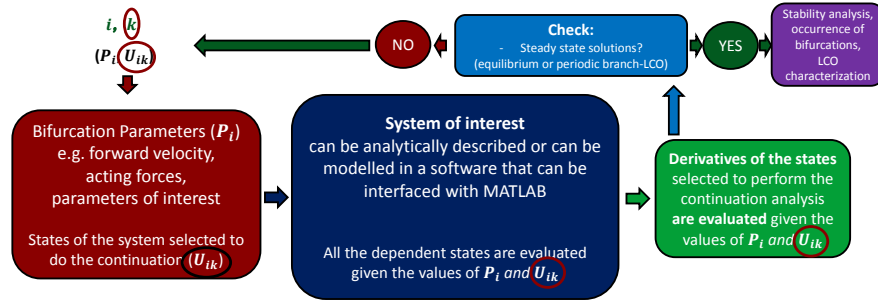


Figure 6: Flow chart that explains how the continuation analysis is performed in Matlab using the Dynamical System Toolbox.<sup>27</sup>

perform bifurcation analysis and AUTO is the used software, in particular the Matlab version of AUTO, the Dynamical System Toolbox.<sup>27</sup> Regarding the multi-body software, LMS Virtual.Lab Motion is the selected one. LMS Virtual.Lab Motion has been interfaced with Matlab using a library available in VLM itself<sup>28</sup> so that all the states of the model are available in the Matlab environment. Figure 7 presents the flow chart of the general process adopted for the coupling. In particular, it is showing the exchange of information occurring between the AUTO version in Matlab and VLM through the Matlab interface available in VLM.

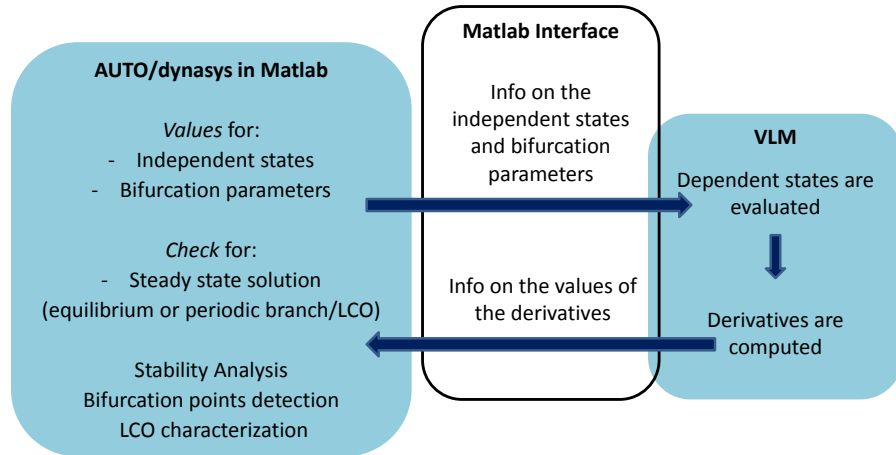


Figure 7: Flow chart describing the exchange of information needed to perform the bifurcation analysis coupling AUTO with VLM using the Dynamical System Toolbox<sup>27</sup> and the Matlab interface available in VLM to ‘handle’ the states of the multi-body model.

## IV. Results

In this section the results obtained performing Sensitivity Analysis and Uncertainty Quantification for the multi-body landing gear system and applying the Iterative Distribution Evolutionary Algorithm (IDEA) are presented. The uncertainty quantification and sensitivity analysis are performed in terms of bifurcation algorithms. Confidence bounds are defined for the possible occurrence of limit cycle oscillations in the presence of parametric uncertainty. These bounds are the loci of the Hopf bifurcation points and that determines subdivision of the interested parameter space identifying the region in which limit cycle oscillations, in particular shimmy, occur.<sup>30–32</sup> The loci of Hopf bifurcation are determined varying the vertical load  $\bar{W}$  acting on the landing gear and the forward velocity  $V$  (Fig. 8). In order to perform the Sensitivity Analysis, the Uncertainty Quantification and then the Optimization, the locii of Hopf bifurcation are suitably discretized as it is shown in Fig. 8. The stated locii of Hopf bifurcation are those obtained for four different sets of parameter values of the landing gear systems that are varied during the SA (table 1).

Parameter	1 <sup>st</sup> set	2 <sup>nd</sup> set	3 <sup>rd</sup> set	4 <sup>th</sup> set
Mass of the main fitting 1 (kg)	9.42	9.95	10.35	9.62
Mass of the main fitting 2 (kg)	5.17	5.10	5.34	4.88
Mass of the main fitting 3 (kg)	71.61	70.20	65.47	70.78
Mass of the wheel axle (kg)	103.69	95.47	105.82	94.76
Mass of the wheel (left and right) (kg)	107.62	102.42	116.73	117.41
Mass of the trail (kg)	1.98	1.92	1.87	1.76
Stiffness fuselage - main fitting 1 (N m rad <sup>-1</sup> )	1.4310 <sup>6</sup>	1.4210 <sup>6</sup>	1.5710 <sup>6</sup>	1.5510 <sup>6</sup>
Stiffness main fitting 1 - main fitting 2 (N m rad <sup>-1</sup> )	1.4810 <sup>6</sup>	1.5710 <sup>6</sup>	1.4110 <sup>6</sup>	1.4710 <sup>6</sup>
Stiffness trail body - wheel axle (N m rad <sup>-1</sup> )	2.3410 <sup>4</sup>	2.6710 <sup>4</sup>	2.6210 <sup>4</sup>	2.3310 <sup>4</sup>
Stiffness trail body - main fitting 3 (N m rad <sup>-1</sup> )	7.3410 <sup>5</sup>	7.6010 <sup>5</sup>	7.7310 <sup>5</sup>	7.9410 <sup>5</sup>
Damping fuselage - main fitting 1 (N m s rad <sup>-1</sup> )	420.78	383.20	418.48	387.57
Damping main fitting 1 - main fitting 2 (N m s rad <sup>-1</sup> )	7.7210 <sup>4</sup>	8.2610 <sup>4</sup>	8.3410 <sup>4</sup>	7.9410 <sup>4</sup>
Damping trail body - wheel axle (N m s rad <sup>-1</sup> )	376.43	372.12	421.27	404.51
Damping trail body - main fitting 3 (N m s)	373.17	380.37	405.08	416.63

Table 1: Four combinations of values adopted to show the variation of the locii of Hopf bifurcation and the relative discretization.

IDEA is applied with the aim of decreasing the probability of occurrence of shimmy during ground manoeuvres. In particular, the loci of interest are the loci of Hopf bifurcation points and the limit state function is the operational trend describing the variation of the vertical load  $\bar{W}$  as the forward velocity  $V$  changes during ground manoeuvres.

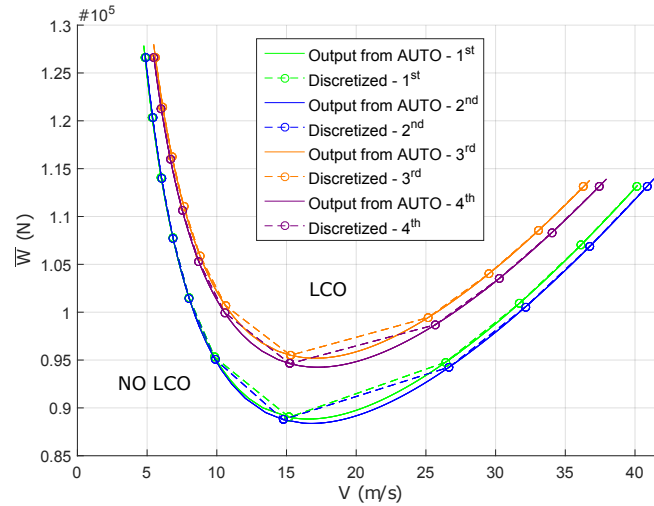


Figure 8: Example of the description adopted for the sought branches to perform both the Sensitivity Analysis and the Uncertainty Quantification for the landing gear model. Each line is a 2-parameter continuation of Hopf bifurcations obtained at a particular set of parameter values (table 1).

### A. Sensitivity analysis and uncertainty quantification of the multi-body landing gear system

The techniques presented by the authors<sup>18–22</sup> have been applied to the computational expensive and complex multi-body landing gear system obtaining very good results. For underlining the significant computational burden, required by bifurcation analyses performed using the multi-body model, table 2 shows the time needed to obtain a single bifurcation branch in the bifurcation diagram in terms of one (the forward velocity)

and two bifurcation parameters (the forward velocity and the vertical load).

Bifurcation Diagram in 1 Parameter	Bifurcation Diagram in 2 Parameters
6 min	20 – 40 min

Table 2: Time required to compute the bifurcation diagrams using a 64-bit Operating System based laptop with a processor Inter(R) Core(TM) i7-4810MQ CPU@2.80GHz and a RAM of 32.0 GB.

To perform SA, main and total effect indexes have been considered and Fig. 9 and 10 show the total Sobol indices adopted to track changes in the shape and translation of locus of Hopf bifurcation (Fig. 8).<sup>23</sup> Table 3 provides the variation considered for each parameter ( $\pm 7\%$ ) together with the adopted distribution.

Parameter	Label	Maximum	Minimum	Units	PDF
Mass of the main fitting 1	$M_{MF_1}$	9.3	10.7	kg	log-uniform
Mass of the main fitting 2	$M_{MF_2}$	4.65	9.3	kg	log-uniform
Mass of the main fitting 3	$M_{MF_3}$	65.1	74.9	kg	log-uniform
Mass of the wheel axle	$M_{WA}$	93	107	kg	log-uniform
Mass of the wheel (left and right)	$M_W$	102.3	117.7	kg	log-uniform
Mass of the trail	$M_T$	1.753	2.017	kg	log-uniform
Stiffness fuselage - main fitting 1	$k_1$	1395000	1605000	N m rad <sup>-1</sup>	log-uniform
Stiffness main fitting 1 - main fitting 2	$k_2$	1395000	1605000	N m rad <sup>-1</sup>	log-uniform
Stiffness trail body - wheel axle	$k_3$	23250	26750	N m rad <sup>-1</sup>	log-uniform
Stiffness trail body - main fitting 3	$k_4$	697500	802500	N m rad <sup>-1</sup>	log-uniform
Damping fuselage - main fitting 1	$d_1$	372	428	N m s rad <sup>-1</sup>	log-uniform
Damping main fitting 1 - main fitting 2	$d_2$	74400	85600	N m s rad <sup>-1</sup>	log-uniform
Damping trail body - wheel axle	$d_3$	372	428	N m s rad <sup>-1</sup>	log-uniform
Damping trail body - main fitting 3	$d_4$	372	428	N m s	log-uniform

Table 3: Parameters and the range of values adopted in the sensitivity analysis and uncertainty quantification

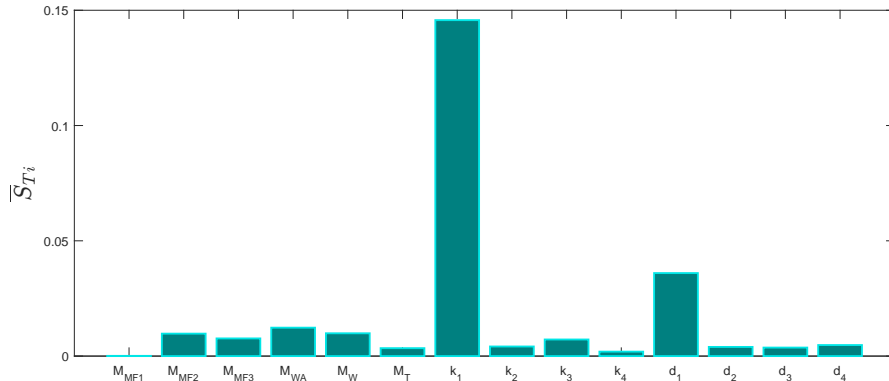


Figure 9: Comparison of the influence of each parameter on the change of shape of the loci considering the total effect  $S_{T_i}$ .

Looking at the results given by the SA,  $k_1$ ,  $k_4$  and  $d_1$  have been selected to then propagate uncertainties in terms of the locus of Hopf bifurcation points adopting the geometrical based method together with the SVD/HOSVD techniques.<sup>18–22</sup> Figure 11 shows the lower and upper confidence bounds obtained in terms of the loci of Hopf bifurcation in dark blue and red. The black points are those belonging to the loci of Hopf bifurcation evaluated using the SVD/HOSVD based method. The other lines are connecting points at the

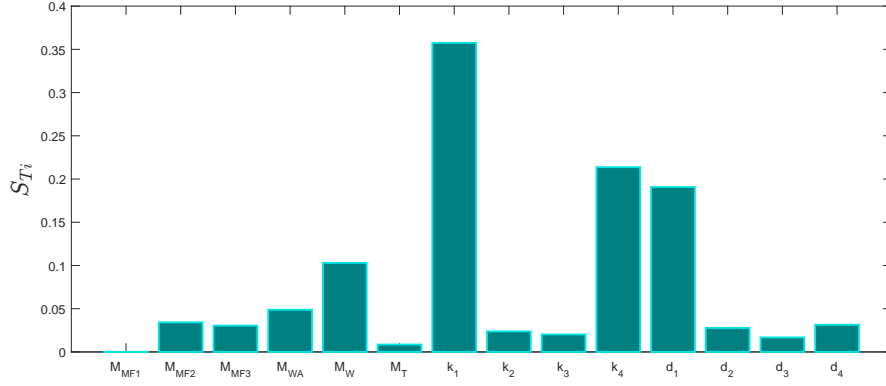


Figure 10: Comparison of the influence of each parameter on the translation of the loci considering the total effect  $S_{T_i}$ .

same quantile, where the quantile increases from a value of 0.1 to 0.9 starting from the lower bound and ending at the upper bound.

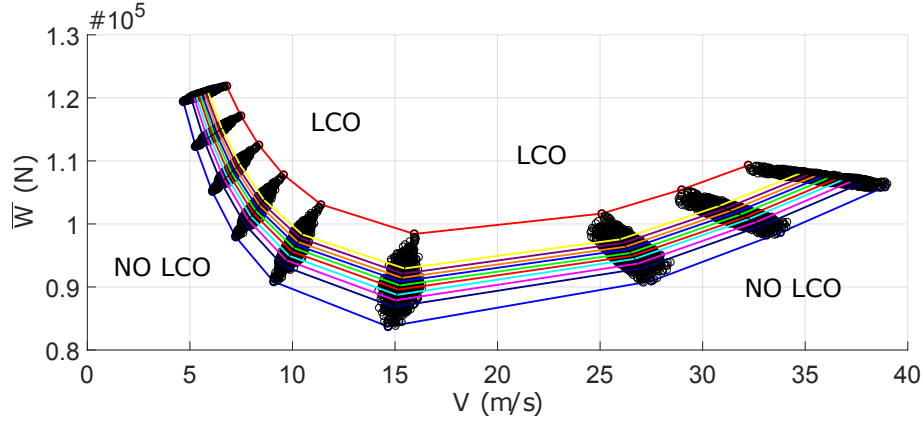


Figure 11: Uncertainty bounds evaluated using the geometric based approach, the HOSVD as the method for feature extraction and a combined side slip condition. The lower and upper confidence bounds are in dark blue and red colors. The black points are some of the points belonging to the loci of Hopf bifurcation evaluated using the SVD/HOSVD based method. The other lines are connecting points at the same quantile, in particular the quantile increases from a value of 0.1 to 0.9 starting from the lower bound and ending to the upper bound.

Looking at the results given by the UQ it is apparent the importance of including the presence of uncertainty during a design process since the influence they have on the response of interest, if an interval approach is considered, the occurrence of shimmy is uncertain in all the operational region between the lower and upper bound in Fig. 11.

The validation of the performed SA and UQ has given very good results with a mean average percentage error that is always less than 1.5%.

## B. Optimization of a landing gear system

The interval of variation, the tolerances adopted in the IDEA and all the coefficients that need to be defined in the Evolutionary Algorithm are defined as.

- the percentage variation  $P_{max_{i_{low}}}$  is fixed equal to  $P_{max_{i_{upp}}}$ . In what it follows, the percentage variation will be then called  $P_{max}$  and is fixed equal to 3.5%.

- The stated tolerances  $Tol_\epsilon$  and  $Tol_p$  are fixed equal to 0.01 and 0.05.
- $min_x$ ,  $max_x$  depend on the range of variation for the parameters.

Latin Hypercube Sampling (LHS) is the technique adopted to define the sampling planes for training the initial surrogate models, while the full factorial design has been adopted to sample the input design factors used in the iterative phase and the dimension is 31.

As previously stated, the optimization of the landing gear has the aim of decreasing the probability of occurrence of shimmy during ground manoeuvres. In particular, this has been addressed by making the probability of intersection between the loci of interest (loci of Hopf bifurcation) and the limit state function (the operational trend) as lower as possible. Figure 12 shows an example of what can occur due to uncertainty in the system.

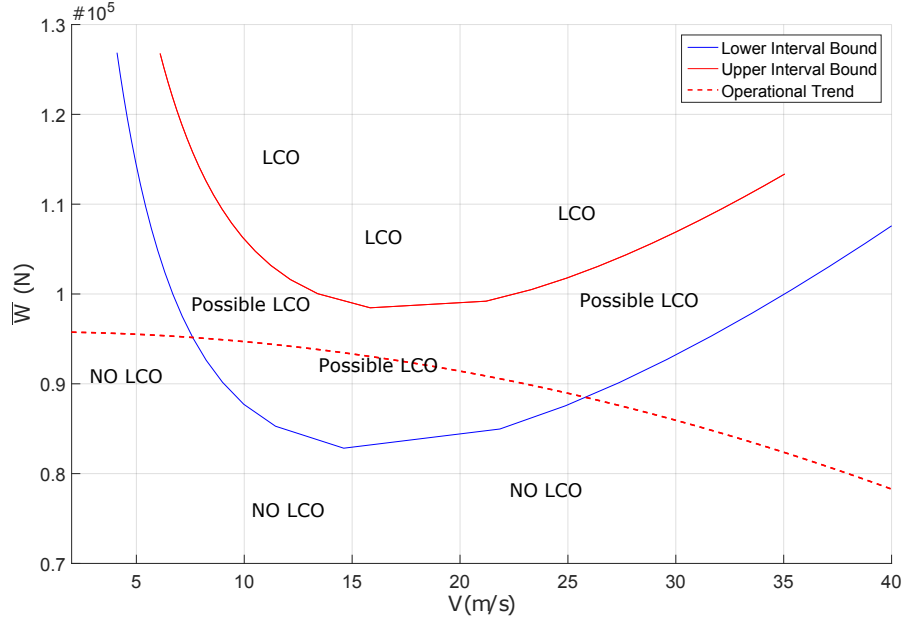


Figure 12: Example of lower and upper confidence bounds for the loci of Hopf bifurcation points and operational trend.

The continues blue and red lines are the lower and upper confidence bounds for the loci of Hopf bifurcation points determined using the SVD/HOSVD based method.<sup>18–22</sup> The dot red line is the operational trend. For all the values of the forward velocity  $V$  between the first and second intersecting point, LCO (shimmy) can occur since the uncertainty in the system. The optimization we are considering has also as aim to acquire an understanding of the acceptable range of uncertainty. The performed application of IDEA considers the nominal values of the uncertain factors as design parameters. The uncertain parameters to be considered are those the much influential to a change of the loci of Hopf bifurcations, i.e. the stability of the system.

The coefficients used to define the limit function trend  $g$  (eq. 1) are first specified (Table 4).

In the first phase the surrogate models evaluated to perform the Uncertainty Quantification, are adopted to identify the set of values  $\mathbf{F}_G^*$  and the interval (eq. 2). Figures 13-15 provide the results related to the Evolutionary phase, the green point stands for  $\mathbf{F}_G^*$ . The hypercubes in the parameter space (in this case a cube) are determined and sorted in an ascending order. Using a full factorial design, the hypercubes are filled with points (red in Fig. 13b) and the QoI ( $\bar{W}$  and  $V$ ) are evaluated for the points inside the initial range of variation for which the surrogate models are trained. Negative points, i.e. those for which the locus of Hopf bifurcation points are intersecting with the operational trend under the considered tolerance, are determined (yellow points in Fig. 14a) and the hypercube are further subdivided as explained in section II. At this step, AUTO and the numerical model are run for all the other sampling points inside the hypercubes that can be further considered since containing the point  $\mathbf{F}_G^*$  and no negative points. Finally, the hypercubes are sorted again and the one with the greater volume is picked out for each considered generation and the optimum sets

parameter	value
$B$	11.04 m
$x_{cg}$	9.5m
$B_m$	$Bm = B - x_{cg}$
$W$	70000 Kg
$\rho$	1.225kg/m <sup>3</sup>
$S \cdot C_L$	128.1306m <sup>2</sup>

Table 4: Parameter values adopted to describe the limit function for the optimization of the mutli-body landing gear design.

are determined (Fig. 14b). A post-processing of the data can be considered and the SVD/HOSVD based method can be applied to eventually propagate the uncertainty for the determined optimum sets (the black points in Fig. 15).

The comparison of the initial confidence bounds and the optimum ones are shown in Figure 16. Moreover, Figure 17 shows a comparison of the loci of quantiles for the initial and optimum confidence bound.

A comparison of PDFs obtained for the initial set of nominal values and the optimum one is shown in Figure 18. Table 5 provides the initial set of nominal/percentage values and the optimum set and relative acceptable uncertainties.

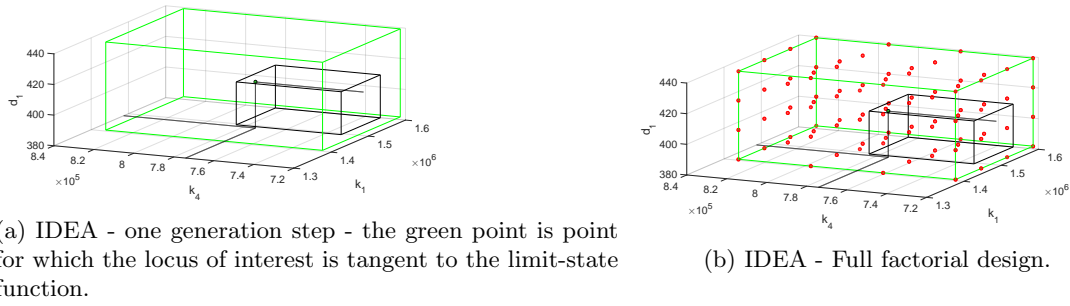


Figure 13

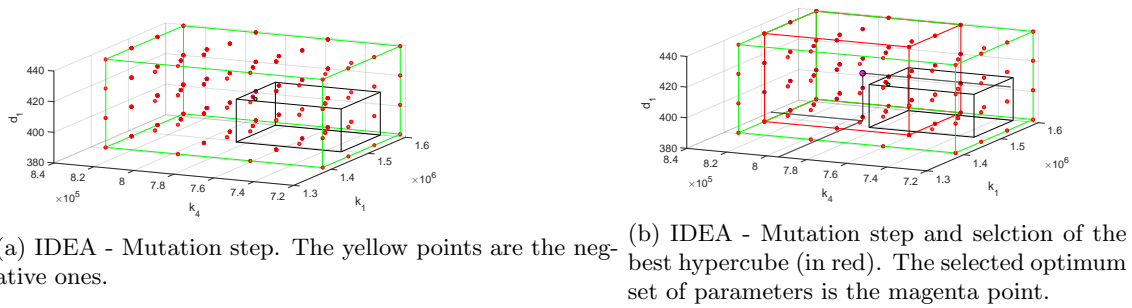


Figure 14

The last result presented here is related to the probability of failure characterizing the system for which the optimum set of values are adopted. The probability of failure is the one along the direction of interest and is related to the points on the locus of Hopf bifurcation that are intersecting the direction of interest and belonging to the region of failure. (Table 6).

The application to the multi body landing gear model is very encouraging for the proposed novel optimization algorithm (I.D.E.A). The most critical probability of failure is decreased assuring an increment in

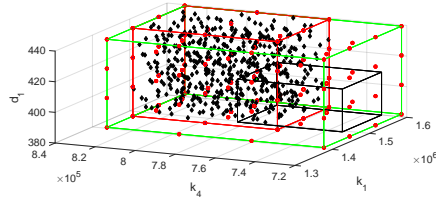


Figure 15: IDEA - Optional postprocessing analysis. The loci are evaluated using the SVD/HOSVD based method at the black points.

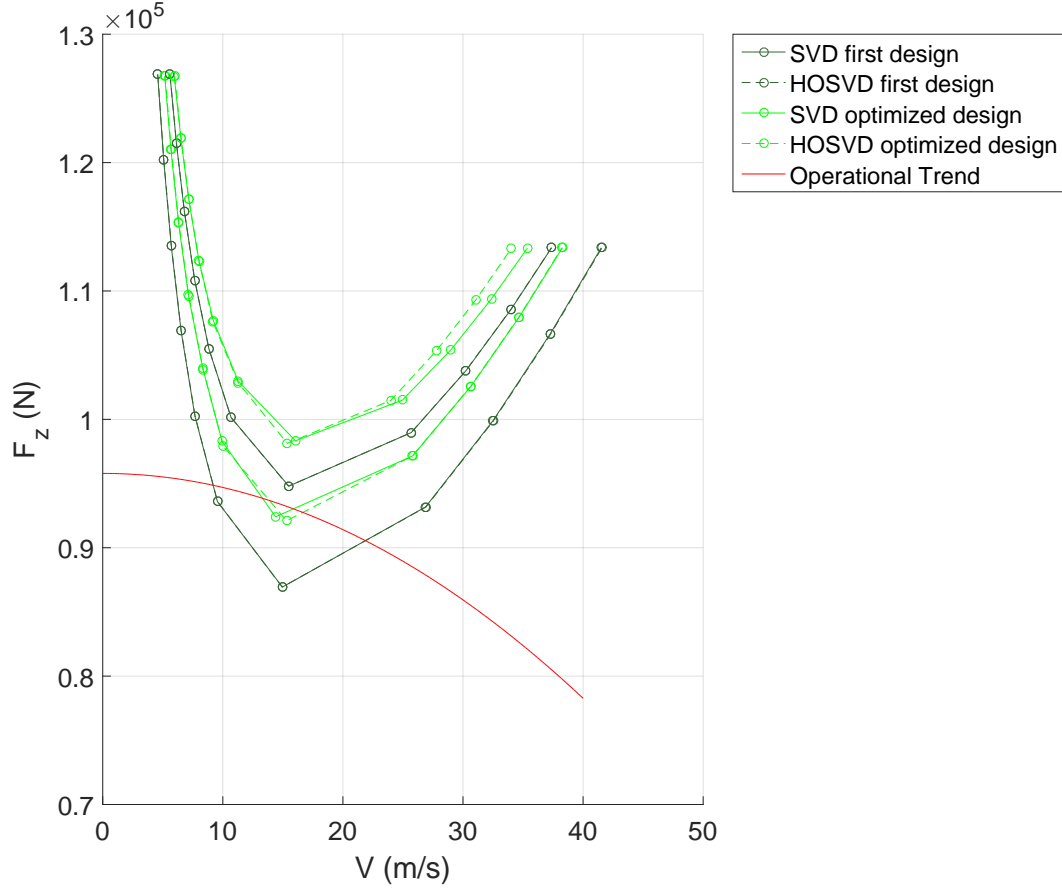


Figure 16: Comparison of the initial confidence bounds and the optimum ones.

	$[c_\psi, I_{psi}, L]$
Starting Point	$[1.5 \cdot 10^6, 750000, 400]$
Optimum	$[1.5257 \cdot 10^6, 794059, 411]$
Initial Uncertainty Range (%)	$[3.5, 3.5, 3.5]$
Acceptable Uncertainty (%)	$[4.55, 4.55, 7]$

Table 5: Results obtained applying IDEA to the multi-body landing gear model.

the structure reliability.



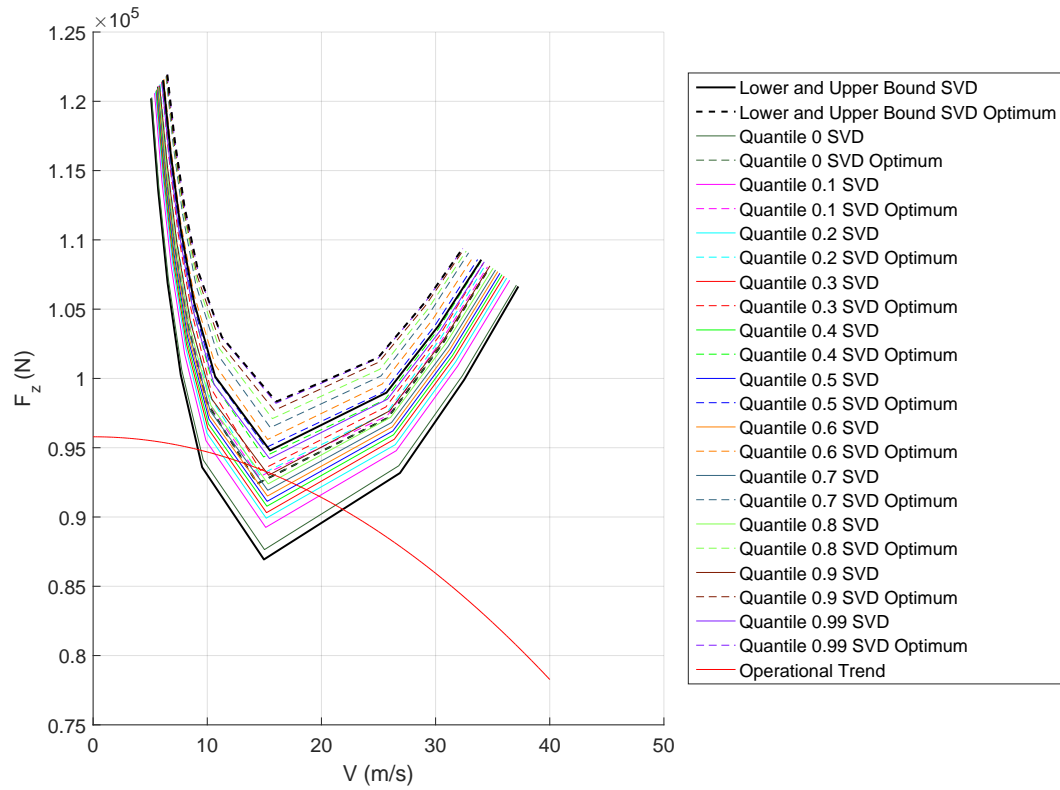


Figure 17: Comparison of the loci of quantile for the initial and optimum confidence bound.

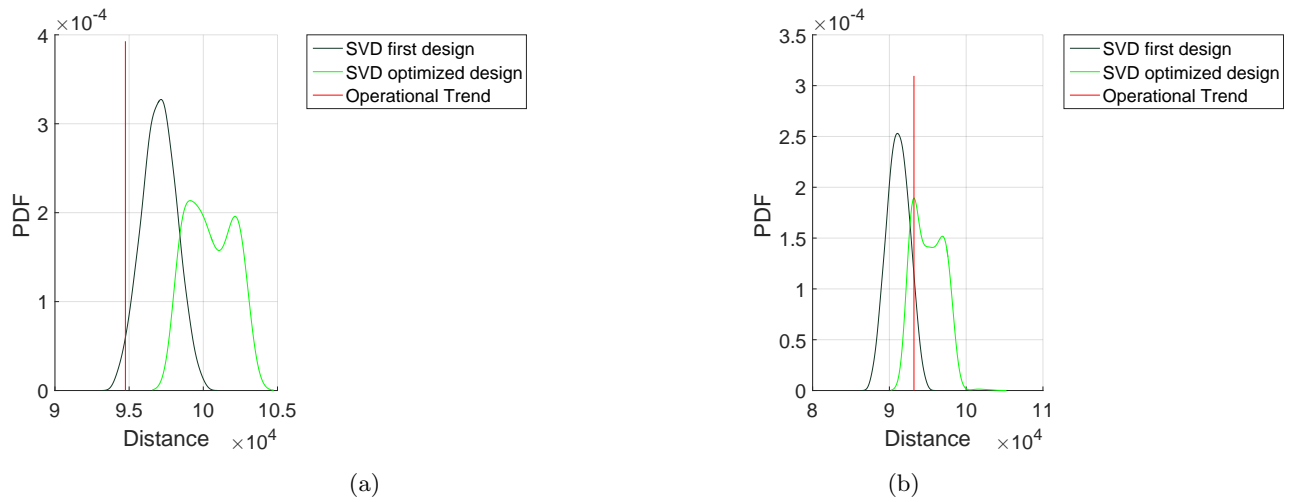


Figure 18: Comparison of PDF obtained for the initial set of nominal values and the optimum one.

Case 1		
Direction of Interest	1	2
SVD		
Starting Design	0.0228	0.9391
Optimized Design	$6.7582 \cdot 10^{-5}$	0.0385
HOSVD		
Starting Design	0.024	0.9339
Optimized Design	$2.2704 \cdot 10^{-4}$	0.151

Table 6: Comparison of probability of failure for the best optimum with the one characterizing the starting landing gear design.

## V. Conclusions

The novel algorithm, Iterative Distribution Evolutionary Algorithm (IDEA), has been adopted to perform optimization under uncertainties minimizing the probability of failure. A reliable optimization has been obtained providing an understanding of the acceptable range of uncertainties and assuring robustness of the system. The validation considered the optimization of landing gear systems, to avoid shimmy phenomena during ground manoeuvre. The results have shown that the proposed method is able of guaranteeing the reduction of probability of occurrence of shimmy phenomena during ground manoeuvre. The obtained results have been obtained reducing the computational time required by numerical simulations thanks to the use of surrogate models and continuation software for stability analysis.

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## References

- <sup>1</sup>Choi, S. K., Grandi, R. V., and Canfield, R. A., editors, *Reliability-based Structural Design*, Springer-Verlag London Limited, 2010.
- <sup>2</sup>Pareto, V., "Manuale di economia politica con una introduzione alla scienza sociale, Piccola biblioteca scientifica, Societ Editrice Libraria," 1906, Milano.
- <sup>3</sup>Mastroddi, F. and Gemma, S., "Analysis of Pareto frontiers for multidisciplinary design optimization of aircraft," *Aerospace Science and Technology*, Vol. 28, No. 1, July 2013, pp. 4055.
- <sup>4</sup>Zeljкови, V. and Maksimovi, S., "Multilevel Optimization Approach Applied to Aircraft Nose Landing Gear," *Scientific-Technical Review*, Vol. LVI, No. 2, 2006.
- <sup>5</sup>Swati, R. F. and Khan, A. A., "Design and Structural Analysis of Weight Optimized Main Landing Gears for UAV under Impact Loading," *Journal of Space Technology Vol. 4, No. 1*, July 2014.
- <sup>6</sup>A. K. Matta, G. V. Kumar, R. V. K., "Design Optimisation Of Landing Gear's Leg For An Un-Manned Aerial Vehicle," *International Journal of Engineering Research and Applications (IJERA)*, Vol. 2, number 4, July-August 2012, pp. 2069-2075.
- <sup>7</sup>Xue, C. J., Dai, J. H., Wei, T., and Liu, B., "Structural Optimization of a Nose Landing Gear Considering its Fatigue Life," *JOURNAL OF AIRCRAFT Vol. 49, No. 1*, January-February 2012.
- <sup>8</sup>Engineering, A., "Multi-Disciplinary Design of an Aircraft Landing Gear with Altair HyperWorks," *Altair HyperWorks*, October 2008.
- <sup>9</sup>Chai, S. T. and Mason, W. H., *Landing Gear Integration in Aircraft Conceptual Design*, September 1996 (rev. March 1997), [http://www.dept.aoe.vt.edu/mason/Mason\\_f/M96SC.html](http://www.dept.aoe.vt.edu/mason/Mason_f/M96SC.html) downloaded in April, 2015.
- <sup>10</sup>Beyer, H.-G. and Sendhoff, B., "Robust optimization A comprehensive survey," *Comput. Methods Appl. Mech. Engrg.*, Vol. 196, 2007.
- <sup>11</sup>Lü, H. and Yu, D., "Stability Optimization of a Disc Brake System with Hybrid Uncertainties for Squeal Reduction," *Hindawi Publishing Corporation Shock and Vibration*, 2016, 13 pages, Article ID 3497468.
- <sup>12</sup>Schëller, G. and Jensen, H., "Computational methods in optimization considering uncertainties An overview," *Comput. Methods Appl. Mech. Engrg.*, Vol. 198, 2008.
- <sup>13</sup>P. Seshadri, P. Constantine, G. I. and Parks, G., "Aggressive design: A density-matching approach for optimization under uncertainty," *Submitted to CMAME, September 26*, 2014.
- <sup>14</sup>Seshadri, P. and Parks, G. T., "Density-Matching for Turbomachinery Optimization Under Uncertainty," *arXiv Preprint arXiv:1510.04162*, 2015.
- <sup>15</sup>Das, S. and Suganthan, P. N., "Differential Evolution: A Survey of the State-of-the-Art," *IEEE Transactions on Evolutionary Computation*, Vol. 15, No. 1, 2011, February.
- <sup>16</sup>Hegerty, B., Hung, C.-C., and Kasprak, K., "A Comparative Study on Differential Evolution and Genetic Algorithms for Some Combinatorial Problems," *1st Workshop on Intelligent Methods in Search and Optimization (WIMSO)*, 2009.
- <sup>17</sup>Swagatam, D., Ajith, A., and Amit, K., *Advances of Computational Intelligence in Industrial Systems*, chap. Particle Swarm Optimization and Differential Evolution Algorithms: Technical Analysis, Applications and Hybridization Perspectives, Springer Berlin Heidelberg, Berlin, Heidelberg, 2008, pp. 1–38.
- <sup>18</sup>Tartaruga, I., Cooper, J. E., Lowenberg, M. H., Sartor, P., Coggon, S., and Lemmens, Y., "Prediction and Uncertainty Propagation of Correlated Time-Varying Quantities using Surrogate Models," *CAES Aeronautical Journal*, DOI 10.1007/s13272-015-0172-1.
- <sup>19</sup>Tartaruga, I., Cooper, J. E., Sartor, P., Lowenberg, M. H., Coggon, S., and Lemmens, Y., "Efficient Prediction and Uncertainty Propagation of Correlated Loads," *SCITECH2015*, January 5-8 2015, Orlando, Florida USA.
- <sup>20</sup>Tartaruga, I., Cooper, J. E., Lowenberg, M. H., Sartor, P., and Lemmens, Y., "Evaluation and Uncertainty Quantification of Bifurcation Diagram: Landing Gear, a case study," *UNCECOMP*, May 25-27 2015, Crete Island Greece.

- <sup>22</sup>Tartaruga, I., Cooper, J. E., Lowenberg, M. H., Sartor, P., and Lemmens, Y., “Geometrical Based Method for the Uncertainty Quantification of Correlated Aircraft Loads,” *ASDJournal* 4(1), 1-20 May 2016..
- <sup>23</sup>Tartaruga, I., Lowenberg, M. H., Cooper, J. E., Sartor, P., and Lemmens, Y., “Bifurcation Analysis of a Nose Landing Gear System,” *SCITECH2016*, January 2016, San Diego, California, USA.
- <sup>24</sup>Worden, K., Staszewski, W. J., and Hensman, J. J., “Natural Computing for Mechanical Systems Research: A Tutorial Overview,” *Mechanical Systems and Signal Processing*, Vol. 25, No. 1, 2011, pp. pp 4–111.
- <sup>25</sup>Sobol’, I. M., “Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates,” *Mathematics and Computers in Simulation*, Vol. 55, 2001, pp. pp 271–280.
- <sup>26</sup>Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S., editors, *Global Sensitivity Analysis, The Primer*, Wiley, 2008.
- <sup>27</sup>dynasys, date accessed November, 2015.
- <sup>28</sup>VLM, “Virtual.Lab Motion,” date accessed June, 2015.
- <sup>29</sup>Tartaruga, I., Lemmens, Y., Sartor, P., Lowenberg, M. H., and Cooper, J. E., “On the influence of longitudinal tyre slip dynamics on aircraft landing gear shimmy,” *ISMA2016 & USD2016 conferences*, September 19-21 2016, Leuven, Belgium.
- <sup>30</sup>Strogatz, S. H., editor, *Nonlinear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry, and Engineering (Studies in Nonlinearity)*, Studies in Nonlinearity, Westview Press, 2nd ed., 2014.
- <sup>31</sup>Guckenheimer, J. and Holmes, P., editors, *Nonlinear Oscillations and Dynamical Systems and Bifurcations of Vector Fields*, Vol. 42 of *Applied Mathematical Sciences*, Springer, 1983.
- <sup>32</sup>Seydel, R., editor, *Practical Bifurcation Stability Analysis*, Interdisciplinary applied Mathematics, Springer, 3rd ed., 2010.
- <sup>33</sup>Doedel, E. and Oldeman, B., editors, *Auto-07p: Continuation and Bifurcation Software*, 2012, <http://www.dam.brown.edu/people/sandsted/auto/auto07p.pdf> downloaded in November, 2014.